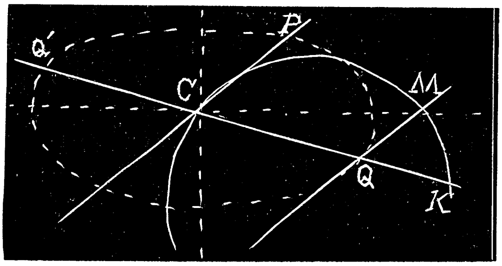

Proof. MN is tangent to the conic.

Now it can be easily shown that if a tangent meets any two lines in such a way that the product of its segments equals the square on the parallel semi-diameter, then the two lines are parallel to a pair of conjugate diameters. In the above construction, $NQ.QM=CQ.QK=CP^2$.

Therefore, CM and CN are the axes (since they are at right angles). Now let r=side of a square whose area is four times the triangle QCP; then $r^2=2ab$. Let s=hypotenuse of a right triangle whose legs are CP and CQ; then $s^2=CP^2+CQ^2=a^2+b^2$. Now construct $m=\sqrt{(s^2+r^2)}$ and $n=\sqrt{(s^2-r^2)}$; then $m=a+b$, $n=a-b$; hence, $a=\frac{m+n}{2}$, $b=\frac{m-n}{2}$.

II. **Solution by H. C. FEEMSTER, York College, York, Nebraska.**

Let $a'a$ and $b'b$ be the conjugate diameters, making the angle aOb. Through a draw a parallel to Ob, this will be a tangent to the ellipse. Extend Oa to P, so that Ob is a mean proportional between Oa and OP. Draw a perpendicular bisector of Op intersecting the tangent through a at C. With C as a center draw a circle through P and O, cutting aC in A and B. Draw OA and OB; these will be the semi-axes of the ellipse, for $Aa\times aB=Oa\times aP=Ob^2$. OA and OB are conjugate diameters, and are perpendicular to each other. And these make the required angles with the given diameters. See Salmon's *Conic Sections,* pages 172 and 173.

Also solved by C. N. Schmall.

CALCULUS.

325. **Proposed by E. B. ESCOTT, Ann Arbor, Michigan.**

$$\text{Integrate } \int \frac{4x^6-a^3}{\sqrt{(x^6-a^3)}}dx.$$

Solution by MARY E. WILSON, Olivet College, Olivet, Michigan.

Let $x^2=z$. Then $dx=\frac{1}{2}\frac{dz}{\sqrt{z}}$. Hence, $\int\frac{4x^6-a^3}{\sqrt{(x^6-a^3)}}=\int\frac{4z^3-a^3}{2\sqrt{(z^4-za^3)}}dz$

$$=\sqrt{(z^4-a^3z)}+C=x\sqrt{(x^6-a^3)}+C.$$

Also solved by G. W. Hartwell, and the Proposer. The Proposer multiplied both numerator and denominator by x, after which the integral is seen directly.